On Distributed Hypothesis Testing with Constant-Bit Communication Constraints

Xiangxiang Xu[1,*] Shao-Lun Huang[1]

ITW 2021 [arXiv:2109.06388]

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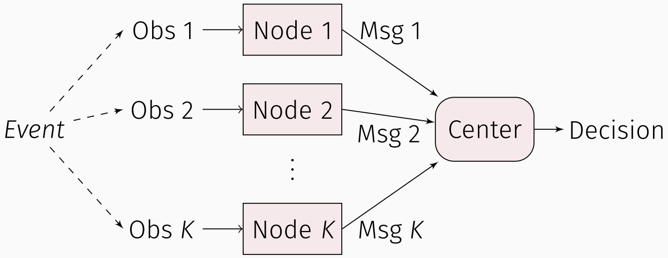
Background: Distributed Inference



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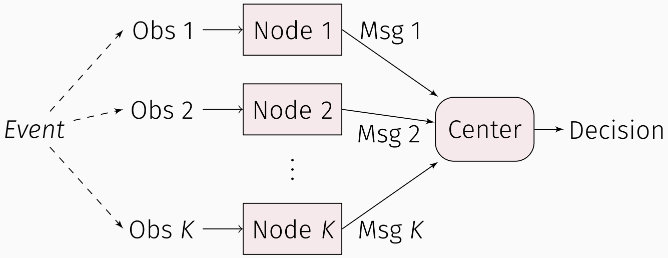

- Distributed Detection

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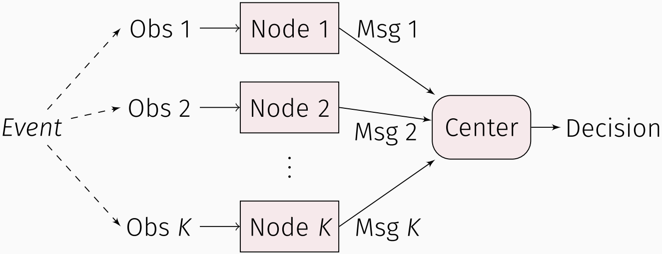

- Distributed Detection
- Sensor Fusion

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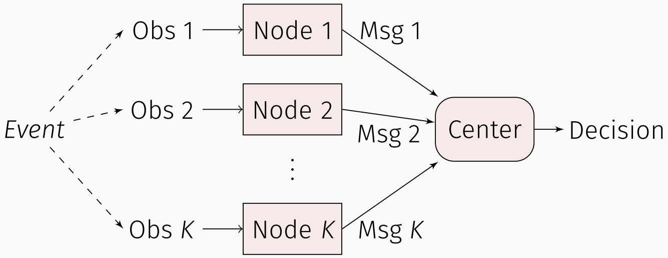
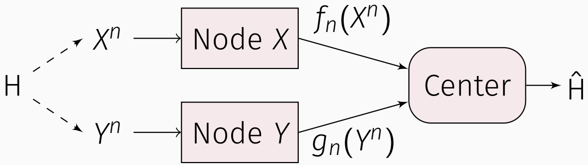


- Distributed Detection
- Sensor Fusion
- Federated Learning

Problem Formulation: Distributed Hypothesis Testing (DHT)



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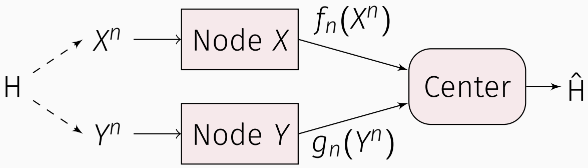
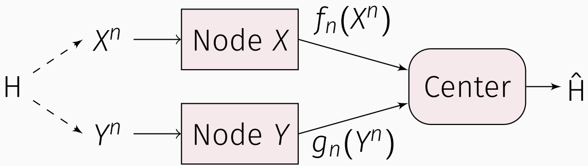


- $\mathrm{H} \in \{0, 1\}$: Binary Hypothesis



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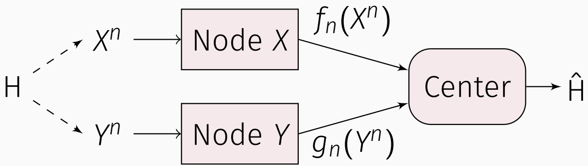

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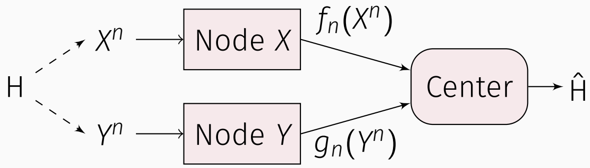
- $f_n(X^n), g_n(Y^n)$: Transmitted Messages



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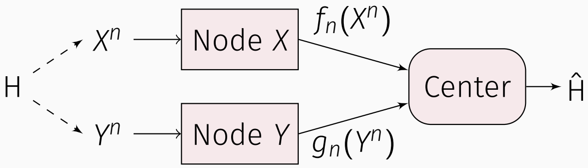

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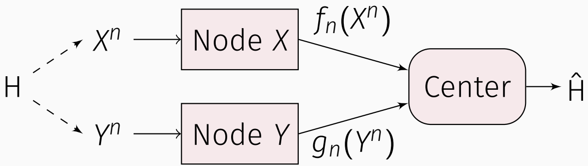

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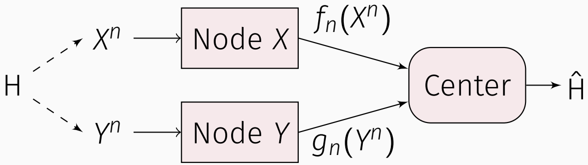
- $\mathrm{dec}(\cdot, \cdot)$: Central Decoder

Communication Constraints in DHT



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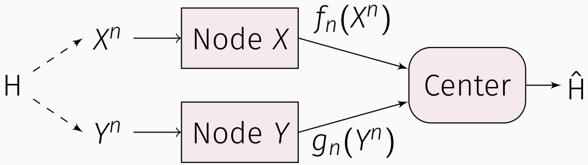
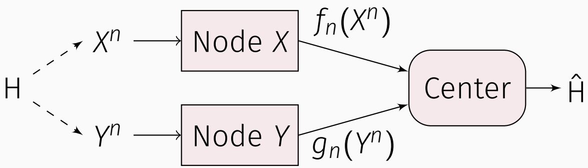


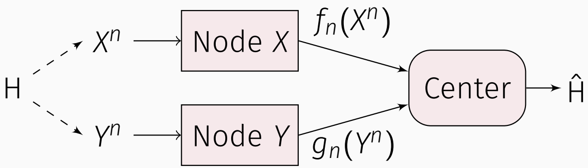
$\|f_n\|$, $\|g_n\|$: sizes of message sets



$\|f_n\|$, $\|g_n\|$: sizes of message sets

Rate Constraints (R_X, R_Y)

$$\|f_n\| \dot{\leq} \exp(nR_X), \|g_n\| \dot{\leq} \exp(nR_Y)$$



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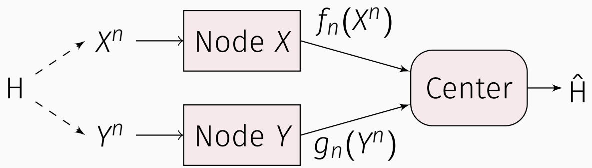
e.g. $(0, 0)$ *zero-rate compression*



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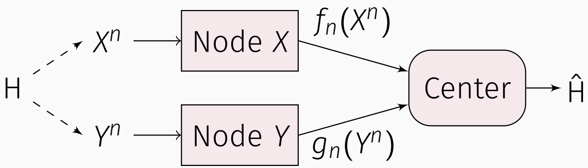
Communication Constraints in DHT



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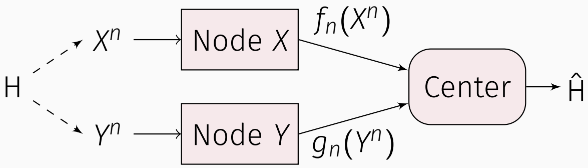
e.g. $(0, 0)$ *zero-rate compression*

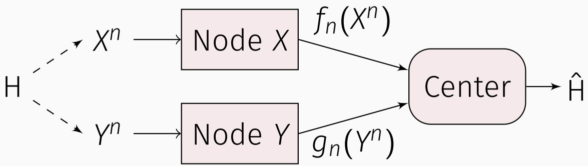
Constant Constraints $(0_{M_X}, 0_{M_Y})$

$$\|f_n\| \leq M_X, \|g_n\| \leq M_Y$$

e.g. $(0_2, 0_2)$ *one-bit compression*

Performance Metrics: Errors and Error Exponents



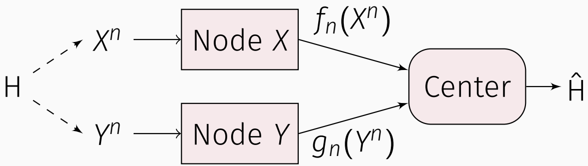


Type-I Error

$$\pi_0 \triangleq \mathbb{P}\left\{\hat{\mathsf{H}} \neq 0 \middle| \mathsf{H} = 0\right\}$$

Type-II Error

$$\pi_1 \triangleq \mathbb{P}\left\{\hat{\mathsf{H}} \neq 1 \middle| \mathsf{H} = 1\right\}$$



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Error exponent E_0: $\pi_0 \doteq \exp(-nE_0)$

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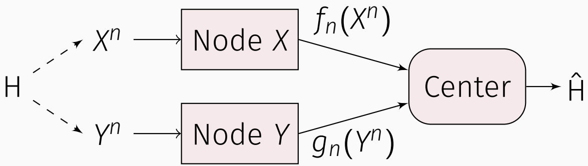

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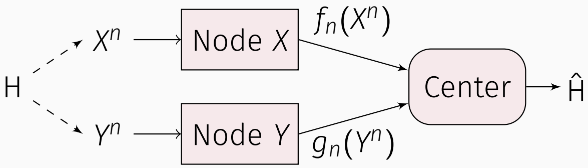
Optimal E_1 under $\pi_0 < \epsilon$



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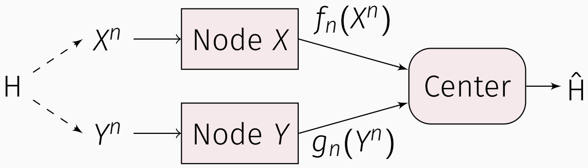
Optimal E_1 under $\pi_0 < \epsilon$ or **Optimal (E_0, E_1) Trade-offs**



Constraints	Opt. E_1 under $\pi_0 < \epsilon$	Opt. (E_0, E_1) Trade-offs
$(R_X, \log \lvert\mathcal{Y}\rvert)$	[Ahlswede & Csiszár, 1986]	
(R_X, R_Y)		
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$(0_2, 0_2)$		
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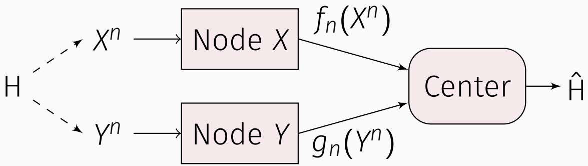

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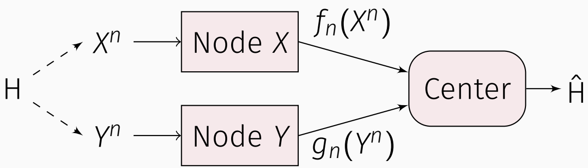
Existing Results



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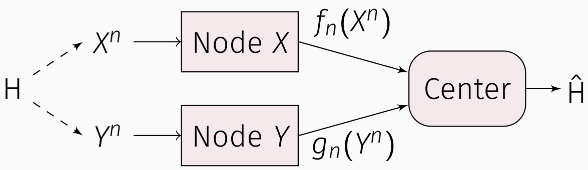

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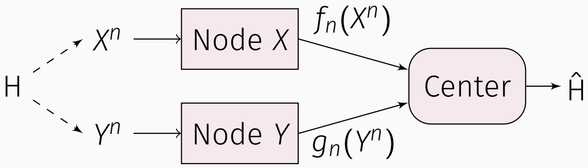

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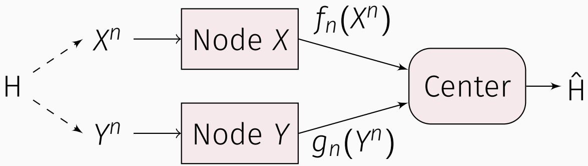

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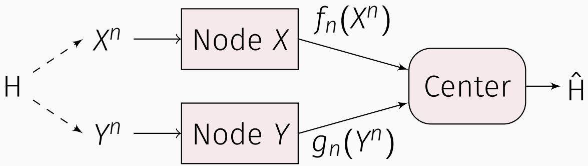

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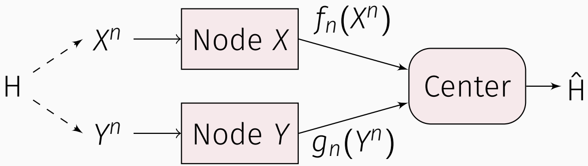

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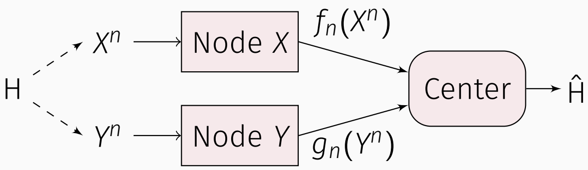

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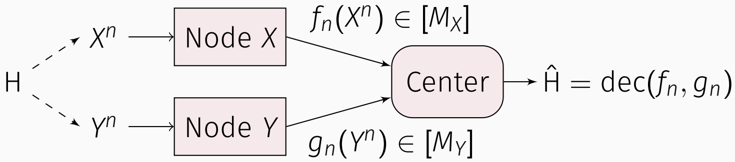
$(0_{M_X}, 0_{M_Y})$ — Constant Communication Bits



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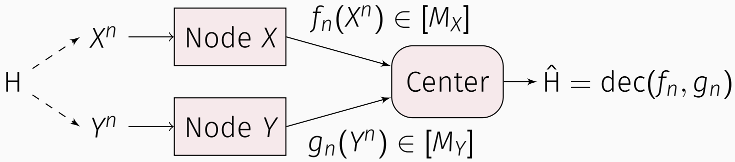
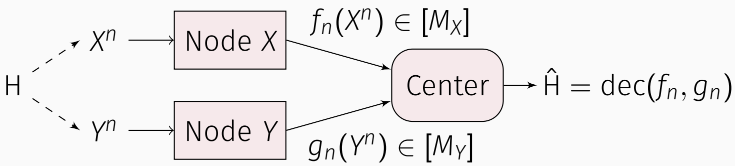


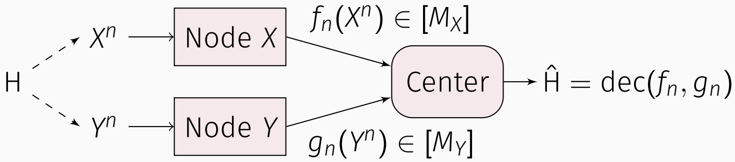
$[M] \triangleq \{0, 1, \ldots, M-1\}$: Message Set with M Symbols



$[M] \triangleq \{0, 1, \ldots, M-1\}$: Message Set with M Symbols

Design of

$(0_{M_X}, 0_{M_Y})$ — Constant Communication Bits



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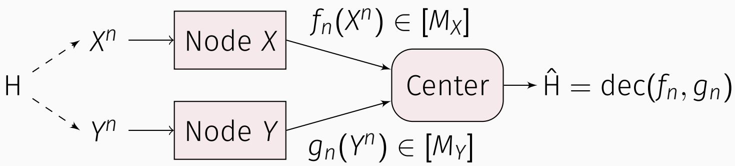

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Design of

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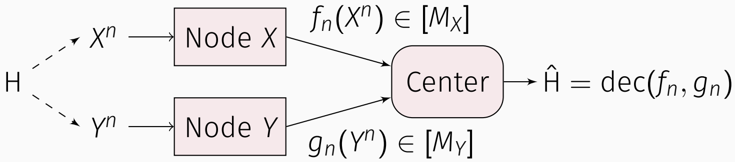
  - $\mathcal{X}^n \to [M_X], \mathcal{Y}^n \to [M_Y]$



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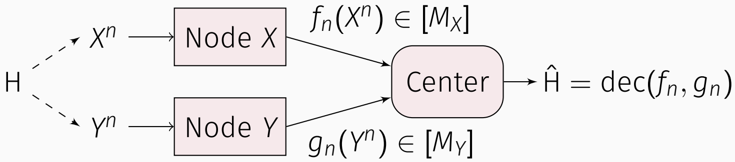

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  - extracting "informative" bits from sequences
- Decoder $\mathrm{dec}(\cdot, \cdot)$

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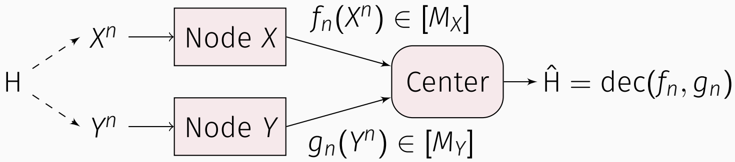

$[M] \triangleq \{0, 1, \ldots, M-1\}$: Message Set with M Symbols

Design of

- Encoders f_n, g_n
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  - extracting "informative" bits from sequences
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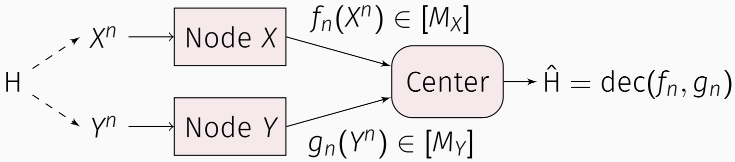

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  - optimal fusion rule

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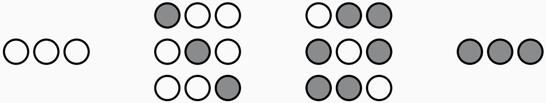
Example: binary sequences of length $n = 3$

Type $\leftrightarrow$ # of ones	0	1	2	3
Sequences				



Encoder Design: Information Extraction from Sequences

- \# of types $=$ poly(n)
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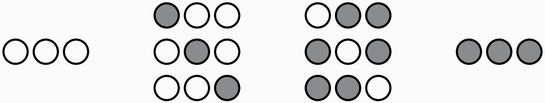

Describing the type is much easier!

Type-based encoders: functions of input type

$$
\begin{aligned}
f_n : x^n &\longmapsto f_n(x^n) \\
g_n : y^n &\longmapsto g_n(y^n)
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Type-encoding Functions

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Optimality (for all decoder choices!)

Given zero-rate encoders f_n, g_n, and a decoder dec, one can construct type-based encoders $\tilde{f}_n, \tilde{g}_n$, such that

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Proof An application of the blowing up lemma.

Decision: Depends only on marginal types $(\hat{P}_{X^n}, \hat{P}_{Y^n})$.

Sequences with marginal types (Q_X, Q_Y)

$$D_i^*(Q_X, Q_Y) \triangleq \min_{\substack{Q_{XY}:\ [Q_{XY}]_X = Q_X \\ [Q_{XY}]_Y = Q_Y}} D(Q_{XY} \| P_{XY}^{(i)})$$

- Associated probability:

$$\mathbb{P}\left\{(\hat{P}_{X^n}, \hat{P}_{Y^n}) = (Q_X, Q_Y) \middle| \mathrm{H} = i\right\} \doteq \exp\left(-n \cdot D_i^*(Q_X, Q_Y)\right)$$

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(E_0, E_1) is achievable, iff

$$\hat{\mathrm{H}}(Q_X, Q_Y) = 0, \quad \text{for all } (Q_X, Q_Y) \in \mathcal{D}_0(E_0)$$
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Geometric Characterization of Error Exponents

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Coding in $\mathcal{P}^{\mathcal{X}} \times \mathcal{P}^{\mathcal{Y}}$

- Encoding

$$\mathcal{P}^{\mathcal{X}} \longrightarrow [M_X], \mathcal{P}^{\mathcal{Y}} \longrightarrow [M_Y]$$

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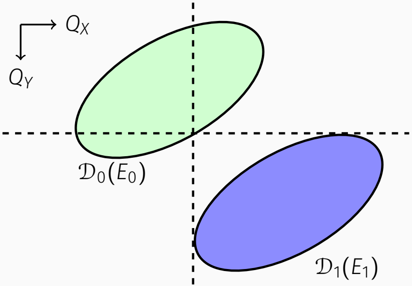

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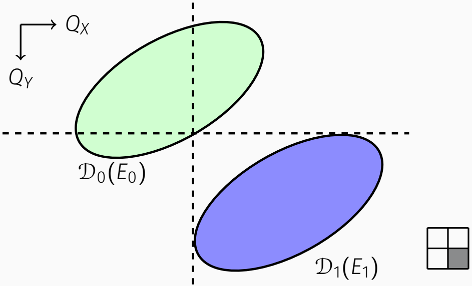

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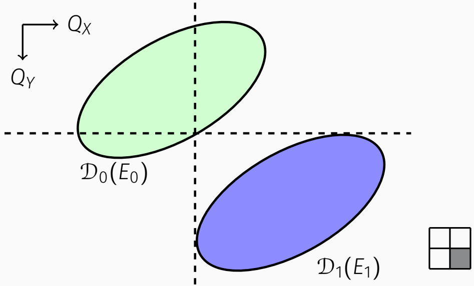

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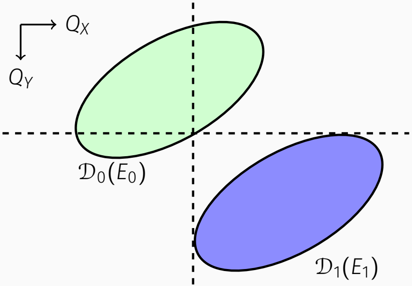

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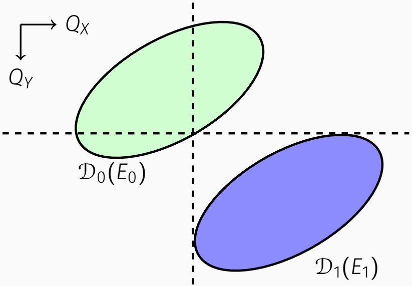

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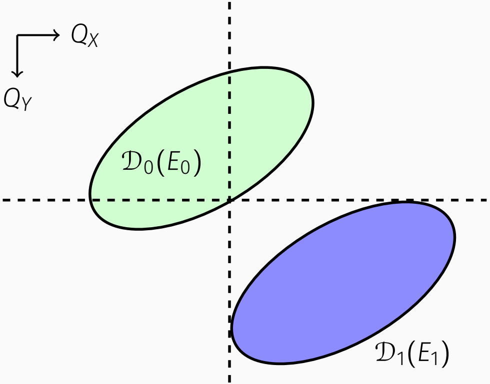
zero-rate compression $(0, 0)$

Add one extra symbol: $(0_2, 0_2) \to (0_3, 0_3)$



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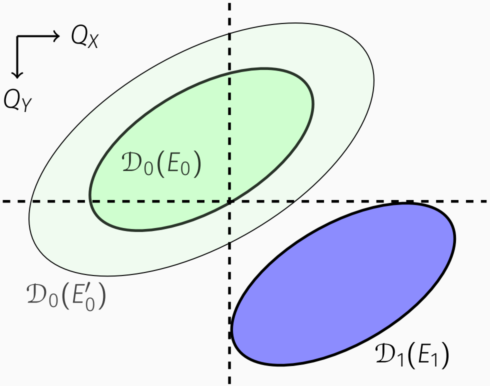

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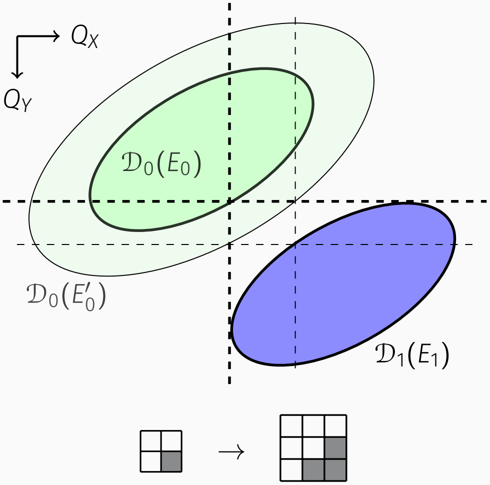

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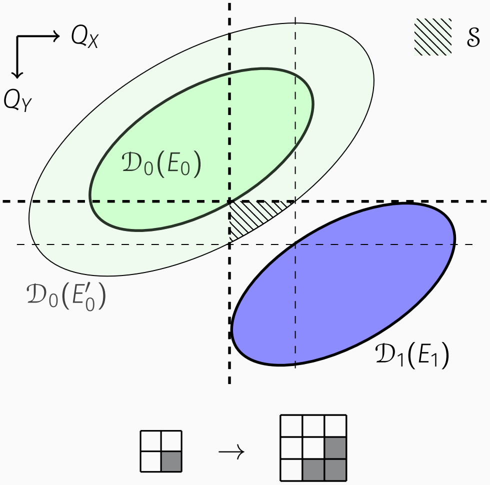

Added symbols: describe $\mathcal{S}$

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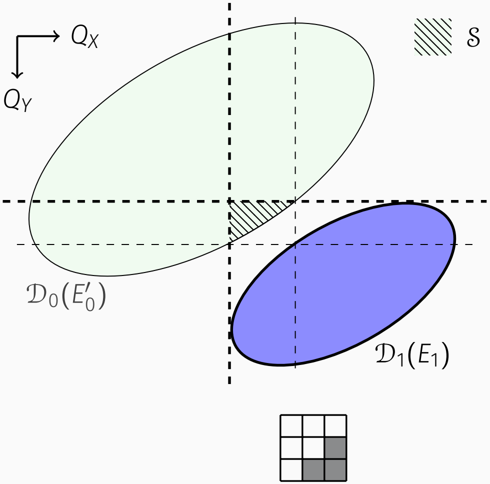

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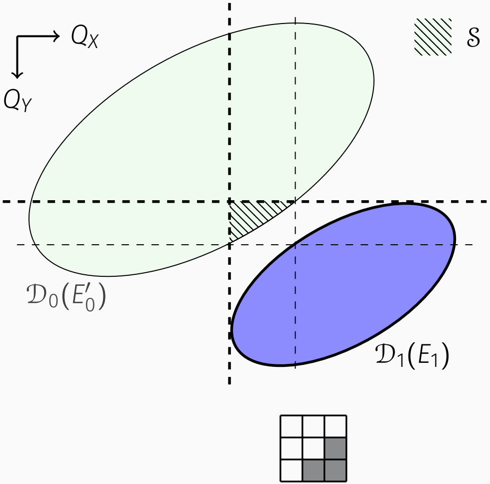
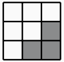


SEPARATE $\mathcal{D}_0(E_0')$ and $\mathcal{D}_1(E_1)$ by



$\Updownarrow$

SEPARATE $\mathcal{S}$ and $\mathcal{D}_1(E_1)$ by

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$$\text{dec}(m_X, m_Y) = \mathbb{1}\{m_X + m_Y \geq M_X \wedge M_Y\}$$

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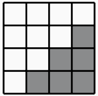
"Reduction" Operations



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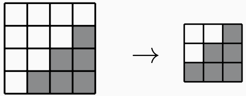

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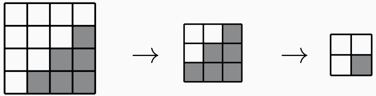
“Reduction” Operations



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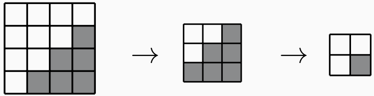

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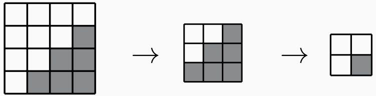

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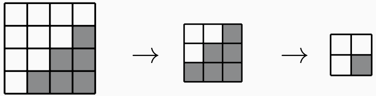

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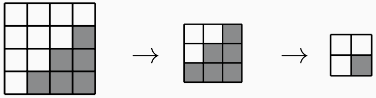

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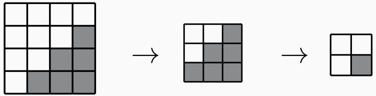

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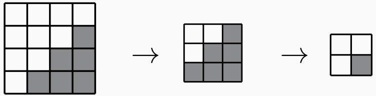

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$\mathbf{min}\{M_X, M_Y\} = 2$

or $(M_X, M_Y) = (3, 3)$

or $P^{(i)}_{XY} = P^{(i)}_X P^{(i)}_Y$ for some $i \in \{0, 1\}$

Numerical Example I

$$\mathcal{X} = \mathcal{Y} = \{0, 1, 2\}$$

$$P_X^{(0)} = P_Y^{(0)} = (1/8, 1/8, 3/4)$$

$$P_X^{(1)} = P_Y^{(1)} = (3/8, 3/8, 1/4)$$

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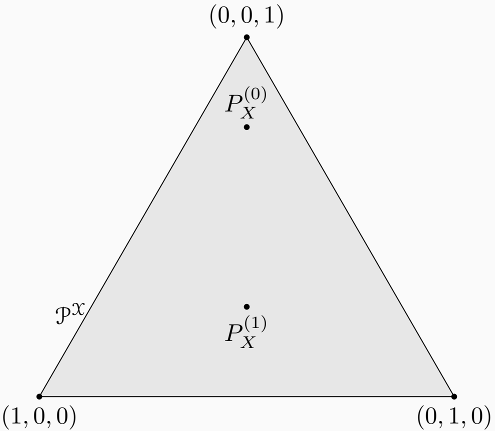

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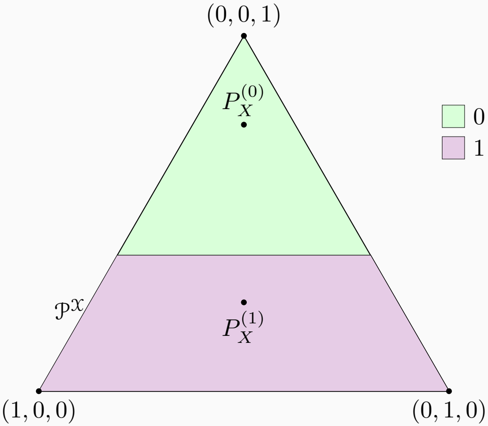
Local Decision at X: Neyman-Pearson



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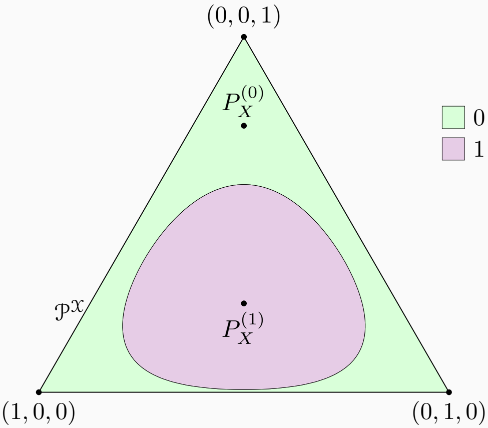
Local Decision at X: Hoeffding (Generalized Likelihood Ratio Test)



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- Comm. Constraints: $(0_7, 0_7)$

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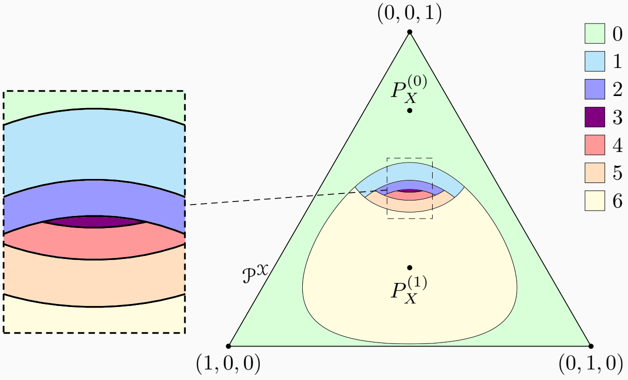
DHT: Type-encoding Function

$$\theta_X : \mathcal{P}^{\mathcal{X}} \to \{0, 1, \ldots, 6\}$$



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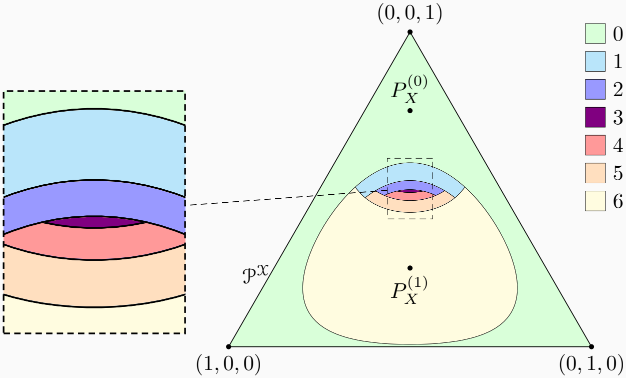

Quantization of LLR is not optimal!

Numerical Example II: Error Exponents

$\mathcal{X} = \mathcal{Y} = \{0, 1\}$

$$P_{XY}^{(0)} = \begin{bmatrix} 1/6 & 1/6 \\ 1/6 & 1/2 \end{bmatrix}, P_{XY}^{(1)} = \begin{bmatrix} 1/2 & 1/6 \\ 1/6 & 1/6 \end{bmatrix}$$

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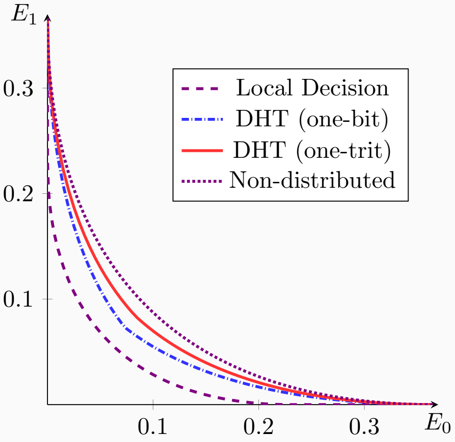

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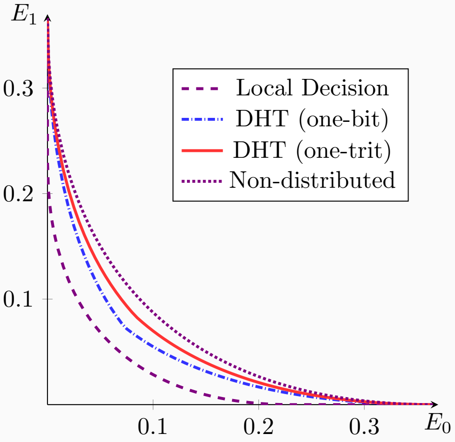

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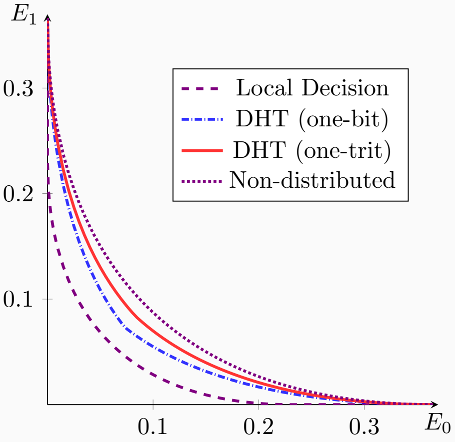

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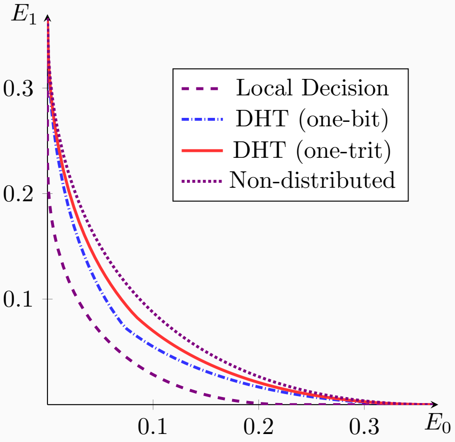

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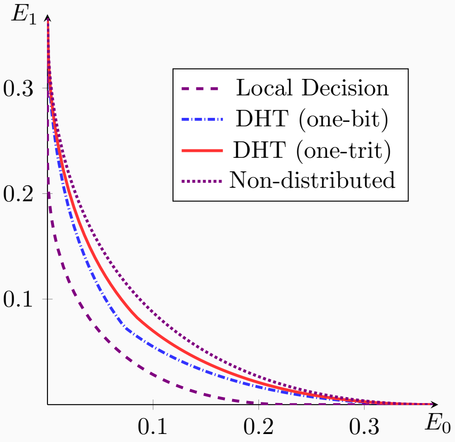
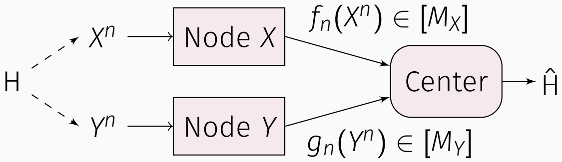
- Non-distributed: $(\log 2, \log 2)$



Summary: DHT with Constant Communication Bits



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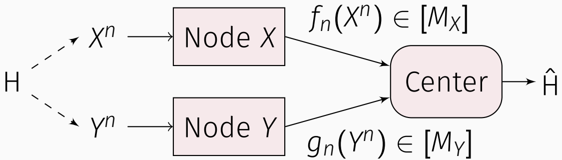
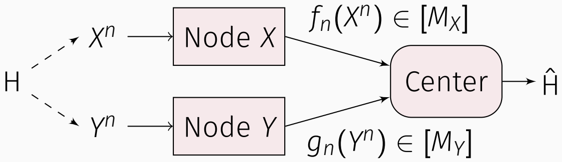
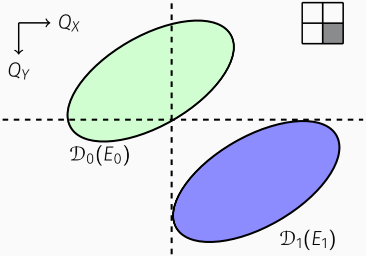


- Encoding over Types is Optimal: Sequence Space $\rightarrow$ Distribution Space





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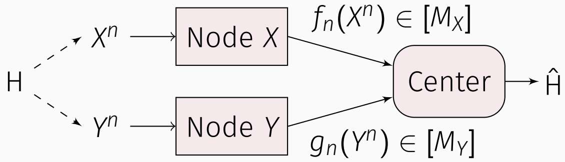
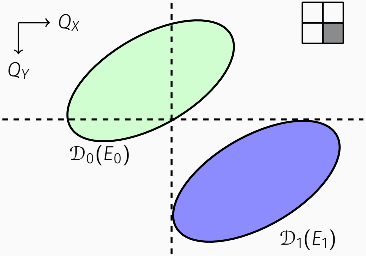
- Geometric Interpretation: Separating Divergence Balls

SUMMARY: DHT with Constant Communication Bits





- Encoding over Types is Optimal: Sequence Space $\rightarrow$ Distribution Space
- Geometric Interpretation: Separating Divergence Balls
- Threshold Decoders: Error Exponents & Optimality